

Limits and Continuity - AP Calculus

Solve each of the following:

$$f(x) = \begin{cases} x^2 - 3x + 9 & , x \leq 2 \\ kx + 1 & , x > 2 \end{cases}$$

1. The function $f(x)$ is defined above. For which of the following values of k , if any, is $f(x)$ continuous at $x = 2$?
A) 1
B) 2
C) 3
D) 4
E) There is no value of k

$$f(x) = \begin{cases} x^2 - 3x, & x \geq 3 \\ \frac{3}{x-3}, & x < 3 \end{cases}$$

2. The function $f(x)$ is defined above. Which of the following statements is true?

A) $\lim_{x \rightarrow 3^+} f(x)$ exists

B) $\lim_{x \rightarrow 3^-} f(x)$ exists

C) f is continuous at $x = 3$

D) f is differentiable at $x = 3$

$$f(x) = \begin{cases} 2x^2 - 5x + 3 & \text{if } x \leq 1 \\ \frac{1}{x-1} + 4 & \text{if } x > 1 \end{cases}$$

3. The function $f(x)$ is defined above. Determine if $f(x)$ is continuous at $x = 1$.

- A) Yes
- B) No, there is a jump discontinuity
- C) No, there is an infinite discontinuity
- D) No, there is a removable discontinuity

4. Let g be the function defined by $g(x) = \frac{(x-3)(x^2-k^2)}{(k-x)(x^2-9)}$, where k is constant. For what value of k , if any, does $\lim_{x \rightarrow 3} g(x) = 2$

A) -3

B) 6

C) -15

D) There is no value of k

5. The function f is defined by $f(x) = \frac{|x| \cdot (\sin(x)\cos(x))}{x^3 - 8}$. At how many values of x does f have a discontinuity?

A) Zero

B) One

C) Two

D) Three

6. If f is a function such that $\lim_{x \rightarrow \infty} h(x) = 0$, which of the following could be $h(x)$?

A) $\sin(x)$

B) $\tan(x) \cdot \cos(x)$

C) $\frac{x}{\ln(x)}$

D) $\frac{5x^2 + 4x - 3}{3x^3 + 2x^2 - 4}$

7. The function h is continuous at $x = 1$.

$$h(1) = \lim_{x \rightarrow 1^-} h(x) + \lim_{x \rightarrow 1^+} h(x) + 1.$$

Which of the following must be true?

A) $h(x)$ is differentiable at $x = 1$

B) $h(1) = -1$

C) $h(1) = 1$

D) $h(1) \neq \lim_{x \rightarrow 1} h(x)$

8. $\lim_{x \rightarrow -5} \frac{x+5}{|x+5|}$ is

A) -1

B) 1

C) 0

D) nonexistent

9. Let f be the function given by $f(x) = \frac{x-1}{5|x-1|}$. Which of the following is true?

A) $\lim_{x \rightarrow 1} f(x) = \frac{1}{5}$

B) $f(x)$ has an infinite discontinuity at $x = 1$

C) $f(x)$ has a removable discontinuity at $x = 1$

D) $f(x)$ has a jump discontinuity at $x = 1$

10. $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^4 + 3x - 2}}{2x^2 - 5x + 1}$ is

A) 2

B) 0

C) 8

D) infinite